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# The scaling law of the pinning force and anomalous Hall effect in the high-$T_c$ superconductors

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**Abstract.** A novel approach to evaluating the pinning force in superconductors is developed on the basis of the extended model of thermally activated depinning. Our method expresses in a unified manner the influence of temperature, magnetic field and current density upon the pinning force acting on the flux. It explains the resistive behaviour, the scaling law of pinning force and the anomalous Hall effect in the mixed state of the superconductors.

## 1. Introduction

During the last few years, attempts to understand vortex motion in high-$T_c$ superconductors (HTSCs) have generated intense interest. From the viewpoint of application, this issue is related to an important technological quantity: the depinning current density $J_c$. Although the classical Anderson–Kim [1] model has been successfully applied for the conventional superconductors for more than two decades, the mixed state of HTSCs, owing to the strong thermal fluctuation and short coherent length, has shown many unconventional and debated properties [2–9].

Very recently a new extended model of vortex dynamics has been suggested [10]. In the present paper we shall use this model to derive the equation for pinning force and current density and to give reasonable explanations for the anomalous Hall effect and some other transport behaviour of the HTSCs.

## 2. The model

The main idea of the extended model is to consider and include viscous dissipation [11] in the process of thermally activated flux motion. So far this dissipation has only been considered in the flux flow state. However, as a mesoscopic system, whenever the flux moves, energy will be dissipated due to the viscous damping force $\eta V$ per unit volume, with $\eta$ the viscous drag coefficient and $V$ the flux velocity. This viscous dissipation $W_{VS}$ during thermally activated flux motion will reduce the hopping probability and contribute to the activation energies. Thus the $E(J)$ characteristic predicted by the conventional Anderson–Kim model should be modified as

$$E(J) = 2v_0 B \exp(-U_0/kT) \exp(-W_{VS}/kT) \sinh(W_{Lorentz}) \tag{1}$$

where the velocity pre-factor $v_0$ contains the mean hopping distance and the attempt frequency of the activated flux ensemble. The temperature, field and current dependences of the conventional activation energy $U_0$ may be approximated by

$$U_0(T, B, J) = U^*(t, b)(1 - x^2) \tag{2}$$

as an attempt to enable further numerical estimations to be made. In equation (2) we have used the reduced variables

$$t \equiv T/T_c(B) \qquad x \equiv J/J_d(T, B) \qquad b \equiv B/B_{c2}. \tag{3}$$

Here $T_c$ and $B_{c2}$ have their usual meanings, $J_d$ is the depairing current density which can be determined from the reversible property λ (penetration depth) and $B_c$ (thermal dynamic critical field) as shown by Tinkham [12].

The viscous contribution to the activation energy is given by

$$W_{VS} = \eta v A = E(J) B A/\rho_f. \tag{4}$$

Here we take Bardeen–Stephen [11] viscous coefficient $\eta = B B_{c2}/\rho_n = B^2/\rho_f$; $\rho_f = \rho_n[(1-b)x + b]$ is a linear interpolation between the Bardeen–Stephen result $\rho_f = \rho_n b$ for low $J$ ($x \to 0$) and $\rho_f = \rho_n$ for high current densities approaching $J_d$ ($x \to 1$), or $B$ approaching $B_{c2}$ ($b \to 1$). $A$ is the product of the moving flux bundle volume and the range of the pinning potential.

The energy due to the Lorentz driving force $JB$ is given by

$$W_{Lorentz} = JBA. \tag{5}$$

We express the electric field in reduced dimensionless units as

$$y \equiv E(J)/\rho_f(T) J_d(T, B) \tag{6}$$

and rewrite the $E(J)$ characteristic in the normalized form

$$y(x) = 2\exp[-u_1(1-x^2)]\exp[-u_2 y]\sinh[u_2 x] \tag{7}$$

with the two dimensionless parameters

$$u_1 = U^*(t, b)/kT \tag{8}$$

$$u_2 = J_d(t, b) B A/kT. \tag{9}$$

The pre-factor $v_0 B/\rho_f J_d$ as given by equation (1) and equation (6) has been included in the function $U_0$.

Equation (7) describes three regimes of flux dynamics, i.e. thermally activated flux flow, creep and flux flow, in a satisfactory way and exhibits qualitatively the same $S$-shaped $E(J)$ characteristics [10] as the experimental results [13]. There is now strong experimental evidence for this model [14]. From the Feymann path-integral approach, $W_{VS}$ in equation (1) is due to the dissipation correction to the undamped effective action, which has been first discussed by Caldeira and Leggett [15] for the quantum tunnelling problem. It is also worth noting that equation (1) has a similar form to the low-friction Brownian motion mobility in a period potential beyond the linear response [16].

## 3. Critical current density and pinning force

On the basis of equation (1), if we define the critical current density $J_c$ by a certain criterion of the electric field $E_c$ as

$$E(J_c) \equiv E_c \tag{10}$$

one finds the equation

$$J_c = J_0\left[1 - \frac{kT}{U_0}\ln\left(\frac{v_0 B}{E_c}\right) + \frac{E_c}{\rho_f J_0}\right] \tag{11}$$

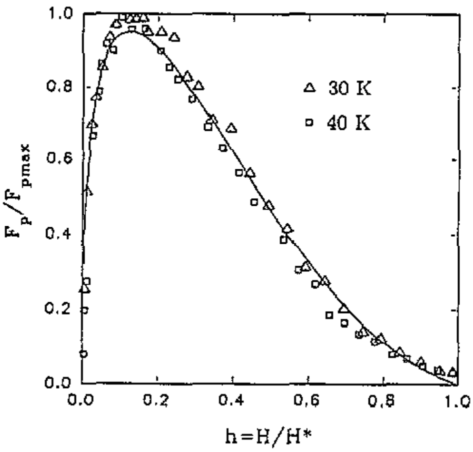


**Figure 1.** The curve fitting by equation (17) to the scaling of pinning force in epitaxial $Bi_2Sr_2Ca_2Cu_3O_x$ thin films reported by Yamasaki *et al* [9].

where $J_0$ is defined as the critical current density where there is no thermal fluctuation:

$$J_0 \equiv U_0/BA. \tag{12}$$

Since $\ln(v_0 B/E_c)$ is a slowly varying function of $B$, we set $\ln(v_0 B/E_c) = \ln(E_0/E_c)$ to be a constant. From equation (11), the irreversibility field $B^*$ due to a certain criterion $E_c$ can be defined as

$$U_0(T, B^*) = kT \ln(E_0/E_c). \tag{13}$$

At $B \approx B^*$, $J_c \approx E_c/\rho_f$, we see a crossover to the depinned flux flow regime. The critical pinning force density $F_p^c \equiv J_c B$ is derived to be

$$F_p^c = J_0 B \left(1 - \frac{U_0(T, B^*)}{U_0(T, B)} + \frac{E_c}{\rho_n}\frac{B_{c2}}{B J_0}\right). \tag{14}$$

If the temperature and field dependences of the activation energy $U_0$ can be expressed separately, i.e.

$$U_0(T, B) = \alpha(T) B^{-p} \tag{15}$$

as reported in some studies [9, 17] and we know that the factor $BA$ in (12) is observed as [13]

$$BA \propto \left(1 - \frac{T}{T^*(B)}\right)^{-3v} \approx \left[1 - \gamma\left(\frac{B}{B^*}\right)^p\right]^{-3v} \tag{16}$$

where $\gamma = \alpha(T)/\alpha[T^*(B)]$ is a slowly varying factor of the order unity; so finally we obtain

$$F_p^C(T) = \alpha(T)\left[1 - \gamma\left(\frac{B}{B^*}\right)^p\right]^{3v}\left(\frac{B}{B^*}\right)^{1-P} B^{*(1-P)}\left[1 - \left(\frac{B}{B^*}\right)^p + \frac{E_c}{J_0\rho_n}\frac{B^*}{B}\frac{B_{c2}}{B^*}\right]. \tag{17}$$

Since both $B^*$ and $B_{c2}$ are functions of only temperature, therefore equation (17) indicates the clear scaling behaviour

$$\frac{F_p^C}{F_{p_{max}}^C} = f\left(\frac{B}{B^*}\right). \tag{18}$$

This kind of pinning force scaling law has been observed in various HTSCs and can be well explained within our model. The curve fitting by equation (17) to the scaling of pinning force in epitaxial $Bi_2Sr_2Ca_2Cu_3O_x$ thin films reported by Yamasaki *et al* [9] is shown in figure 1; our model fits the experimental results quite well.

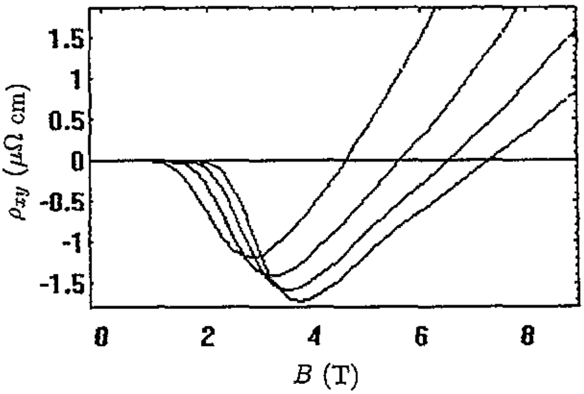


**Figure 2.** Schematic drawing for isothermal Hall resistivity of YBCO as a function of $B$ (equation (21)). From left to right, $T = 88$ K, 86 K, 84 K and 82 K.

## 4. Anomalous Hall effect

Recently, transport measurements on some HTSCs have shown an anomalous behaviour of the Hall resistivity $\rho_{xy}$ [7, 8]. Wang and Ting [18] tried to explain the sign change of $\rho_{xy}$ in low magnetic fields at temperatures near and below $T_c$ by considering the back-flow current due to both pinning forces and other vortices. However, no realistic form of pinning force has been given yet.

Equation (1) of the extended model provides a new basis for more thorough and realistic evaluation of the pinning force in a steady condition. When $T$ is slightly below $T_c$, we expect the small pinning effect and the pre-factor $v_0 B \approx \rho_f J$ to disappear as argued in [19]. Thus, equation (1) can be reformulated as

$$\rho_{xx} = E(T, B, J)/J = \rho_n(B/B_{c2}) \exp[-U_0/kT] \exp[-W_{VS}/kT] \sinh[W_L/kT]. \tag{19}$$

From the force-balance condition of steady flow, one obtains the average pinning force density in the Hall resistivity measurements as

$$\overline{F_p} = JB\left(1 - \frac{\rho_{xx}}{\rho_n}\frac{B_{c2}}{B}\right). \tag{20}$$

By using equation (20) together with the equation (3.13) in the paper by Wang and Ting [18] and setting $\gamma \approx 1$, $\delta \approx 0$, $\chi \approx 1$, we get the following equation for the Hall resistivity:

$$\rho_{xy} = \rho_{xx} B_{c2}\frac{e\tau}{m}\left(\frac{\rho_{xx}}{\rho_n} - 2 + 2\frac{\rho_{xx}}{\rho_n}\frac{B_{c2}}{B}\right). \tag{21}$$

In equation (21), the negative contribution in the large parentheses is always $-2$ and the positive terms are proportional to $\rho_{xx}$. Therefore, we always see a negative sign in the isothermal Hall $\rho_{xy} \sim B$ measurements of the superconducting mixed state at small $B$

where $\rho_{xx}$ is sufficiently small. It becomes positive at larger $B$ and the sign-reversal field $B_{SR}$ can be determined from equation (21) as

$$B_{SR} = 2\frac{\rho_{xx} B_{c2}}{\rho_n(2 - \rho_{xx}/\rho_n)}. \tag{22}$$

A schematic diagram for $\rho_{xy}$ as the function of magnetic field $B$ due to equation (21) is shown in figure 2 which is in quantitative agreement with recent experimental measurements.

## 5. Conclusion

On the basis of the extended model for thermally activated flux motion in the mixed state we obtain new equations for the critical pinning force due to a certain voltage criterion and the average pinning force in transport measurements. The former clearly shows a universal scaling behaviour of the form $F_p^c/F_{p_{max}}^c = f(B/B^*)$ and the latter explains reasonably well the sign-reversal Hall effect at $T < T_c$.

## Acknowledgments

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